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Quickly and accurately predicting horizontal deformations under an embankment using an Artificial Neural Network

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ABSTRACT:

Horizontal deformations can cause damage to objects adjacent to a newly constructed embankment. A tool has been developed to determine in an early state of a project if such a risk exists. The basis of this tool is an Artificial Neural Network (ANN). ANNs are particularly suited to find a relationship between an input and an output parameter, even if it is a very complex one. With this in mind, a network has been trained using calculations with the Finite Element Method (FEM) to find the relationship between the maximum vertical and maximum horizontal displacement under an embankment.

Clearly, only the total vertical displacement is not enough information to predict the horizontal deformations. To make a good prediction of the horizontal deformation one needs, for a completely drained situation, at least two extra parameters: the thickness of the soft soil layers and the height of the embankment. Given these, an ANN can reproduce the finite element result with an error less than 1 percent.

To make a good prediction of the undrained case one needs two extra input parameters: the initial height of the embankment and the initial excess pore water pressure due to this previous load. Again, these five parameters reproduce the results of the FEM analysis within 1 percent. This readily accessible program has the same quality as a complete finite element analysis but is very fast and simple to use.

1 Introduction

The development of the ANN to predict horizontal deformations underneath an embankment is part of a project in which knowledge and tools are developed that enable a planner or designer to use all possible geotechnical knowledge (measurements, simple calculations, complex constitutive models, expert judgments) in an early stage of a project. If such a tool can be made, a planner will have much more knowledge of the financial consequences of his decisions. On top of that, a designer does not need to make adjustments because of bad decisions made early in the project: he can focus solely on the optimal design. This project has so far only focused on horizontal stresses and deformations. If successful, an extension will be made into micropiles or other structural elements related to soil mechanics.

To use all knowledge in a very early stage is an unattainable goal: compromises need to be made. Only a limited amount of data is available; maybe only CPT tests and a few borings. The project should focus on this limited amount of data, in combination with different forms of Artificial Intelligence (AI). The focus of the project will lay on the use of AI within geotechnical engineering. This project will not produce new AI techniques nor new geotechnical theories. The aim is to use AI in a correct, useful and innovative manner within the field of geotechnical engineering.

2 On the use of artificial intelligence

Since the beginning of soil mechanics, people have used mathematical functions to describe the deformation of soil. For example, when deformations are plotted as a function of stresses, a logarithmic scale is often used to help to clarify the behaviour. As science progresses, more and more assumptions regarding the behaviour of soil are added in order to get a workable framework.

One problem is, though, that soil does not necessarily follow these rules. It follows its own laws and does not care about logarithms and parameters we assign it to have.

ANN's, described by Bishop (1995), can be used as a curve fitting tool to follow the constitutive laws people have set up regarding soil mechanics. The neural networks can be trained to produce the same results. This might increase the accessibility of these constitutive laws and make it easier to integrate in the design process. At the very best, though, they might produce the same quality of the results as we have designed in the first place.

A much more powerful application would be to let the ANN follow the laws of the soil itself, and not the laws we assume the soil to have. If, instead of the constitutive laws, data is used to train the network, this behaviour can be reproduced in other cases. This way there is no need to write constitutive laws, but only mathematical techniques are used to model the soil.

Many difficulties arise when one tries to use an ANN for this purpose. Soil has extremely complex behaviour: compression, extension, consolidation, creep, sheer etc. Material models like the soft-soil creep model account for all of these loads to produce deformations accordingly. The one-dimensional formulation of soft soil creep is extremely complex and almost impossible to follow for an ANN. Let alone for the more-dimensional case. For every type of loading, the right test needs to be used in order to train a part of the network. If one aspect is dominating, it is very efficient to train an ANN with this dominating aspect to make a quick prediction. It will remain very difficult to have sufficient, well spread data, to train such a network from project related measurements because projects are often executed in a similar way and measurements are very expensive.

In the context of this project some exercises are designed to show a proper way of using artificial intelligence. The method to predict horizontal deformations as proposed in this paper is the result of one of these exercises. It has been fully reported in Van der Meij (2007).

3 The relationships

When soil is considered to be incompressible, the volume of soil displaced by the settlement of the embankment (V_v) must be equal to the volume of soil displaced horizontally through the incompressible layer (V_h). See the inlay of Figure 1.

The maximum vertical displacement versus the maximum horizontal displacement is, even in the undrained case mentioned before, not a one to one relationship because the shape of the displaced volume can differ. An ANN can be a good tool to find a relationship between these two parameters because it can handle non-linear relationships very well.

The figure below shows the line where the volume of the horizontal displaced ground is equal to the volume of the vertically displaced ground. It is called the undrained response, because in an undrained situation no volume changes are possible.

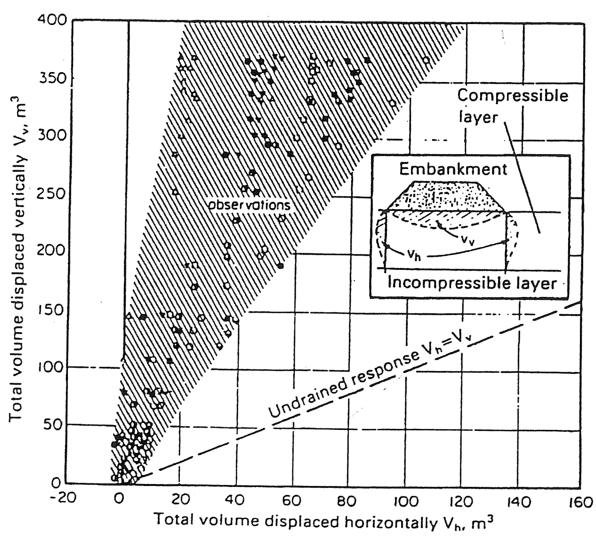


Figure 1: relationship between displaced horizontal volume and vertical volume by Leroueil (1990)

When compaction is taken into account, less volume is displaced horizontally than vertically due to volume changes of the soil. This can be seen in Figure 1: the shaded band lies above the undrained response. The drained response is not such a nice relationship as the undrained one because the geometrical- and material properties start to play a dominant role in the deformations.

First, a basic set of input parameters is derived as an absolute minimum set for training a neural network. The goal of this section is to derive which input parameters are required to predict horizontal displacements, U_x. For the undrained case, $\nu = 0.5$, the following equations hold when examining the inlay in Figure 1

$$V_h = V_v \tag{1}$$

And, just looking at the displacements,

$$u_x = f(u_y) \tag{2}$$

The function f in Equation (2) is a rather complex one, based, for example, on the shape of the settlement trough and the shape of the horizontal deformations. Fortunately, this complexity can be overcome because a neural network can deal with such complex relationships.

In case of compressible soil, if $\nu < 0.5$, one can say that:

$$V_h = V_v - \Delta V \tag{3}$$

ΔV, the volume change, is caused by the compaction of the soft soil. It is a function of a stiffness, E. This E is a function of the stresses and strains.

$$\Delta V_v = f(E) \tag{4}$$
$$E = f(\sigma, \varepsilon) \tag{5}$$

Assuming an embankment is always made out of sand with the same density, the additional stresses are only a function of the raise in height of the embankment.

$$\Delta\sigma = f(\Delta h) \tag{6}$$

Therefore, substituting Equations (5) and (6) into Equation (4) shows the vertical volume change is only a function of Δh and the strain, ε. With strain defined as u_y divided by de layer thickness (d), the vertical volume change is a function of the following parameters:

$$\Delta V_v = f(\Delta h, u_y, d) \tag{7}$$

Substituting in Equation (3) gives

$$V_h = V_v - f(\Delta h, u_y, d) \tag{8}$$

And, assuming that displaced volumes are proportional to the absolute displacements, one can state as well that

$$u_x = f(u_y) - f(\Delta h, u_y, d) \tag{9}$$

Which leaves the horizontal displacements only as a function of three variables.

$$u_x = f(\Delta h, u_y, d) \tag{10}$$

As a first step, it will be investigated if it is possible to make a prediction of the horizontal deformations underneath an embankment if only the raise in height, the settlement of the embankment and the thickness of the soft soil layers is known.

4 Assimilation of data to train the network

For now, it is assumed that the parameters in Equation (10) form a relationship that is sufficient to train a simple ANN to make a prediction of the horizontal deformations. A dataset needs to be available with u_x, u_y, Δh and d, in order to train this network. This dataset can be obtained from measurements or from calculations.

4.1 Measurements

As explained in Section 2, measurements are the ideal form of data to train an ANN. With the proposed dependencies, every inclinometer only gives one measurement. If all three parameters need four points in space to describe their relationship properly, 64 inclinometers need to be installed and work properly in fundamentally different projects. Within the scope of this project this is not realistic, especially if one takes into account that one can expect more parameters like the OCR or the factor of safety to have a significant influence. The number of 64 will increase exponentially. In practice, it is not expected that one can train an ANN with less than 1000 inclinometers, which is not realistic in the short term given the cost of a single inclinometer. Another disadvantage of the use of measurements is that the different embankments will always be around the same factor of safety. They are built as quickly as possible, which means close the minimum legal safety factor. The network will then only be able to predict deformations with a safety factor around the same minimum. If such an inherent property is known, it can be dealt with, but in reality one is never sure when the project data is extrapolated or interpolated. Extrapolation in the case of an ANN will not yield reliable results.

4.2 Calculations

The disadvantage of using calculations to train an ANN is that the quality of the prediction will never be better than the quality of the calculation. Issues like the choice of calculation method and material model become important and will steer the results of the network. Many rules of the geotechnical engineering become implicitly part of the network as an engineer makes the calculation. This is rather unfortunate, because one of the reasons to use ANN's was that mathematical functions do not rely on human assumptions any more.

This disadvantage is not unacceptable because having a properly functioning ANN based on state of the art finite element calculations still means a great improvement over the current practice:

- Analytical methods are often based on linear elasticity and they form the current status quo. A more advanced material model will be a significant improvement.
- The engineering practice that will implicitly become integrated in the network is based on generations of experience and has been validated very thoroughly.
- A finite element analysis is, especially in an early stage of a project, too complex to perform. Having the results of a finite element calculation earlier in a project will be a great improvement.
- In an early stage of a project, not all information is known to make a FEM calculation.
- The calculation time will be reduced from one day to a few seconds.

When preparing a dataset to train an ANN using finite element calculations, one needs to make sure the dataset is representative over all relevant variables. To make the dataset representative over the height of the embankment and the thickness of the soft soil layers is easy for only the right geometries need to be supplied. The input parameter Uy is different because it is a function of the chosen stiffness parameters and the height of the embankment. Using one-dimensional calculations, combinations of E and h need to be found in such a way that the entire desired spectrum of Uy's is covered.

4.3 Conclusions

Given the two options of training an ANN, it is clear that the only way of obtaining a proper dataset is by making sufficient calculations. In the next section, the input space will be examined in order to run the right calculations.

5 Parameters in the FEM analysis

In the previous sections, it has been decided which parameters need to be variable. To make a complete FEM analysis, many other parameters will need to be kept constant. The material model "hardening soil" has been chosen because it gives reasonable predictions of the horizontal deformations and it is very well validated and verified, e.g. by Van der Ham (2007). The lack of time dependency is, for now, also seen as an advantage.

In the left column of Table 1, the variables in the hardening soil model are listed. Most parameters are fixed, others, as decided in the previous sections, will be chosen variable. These variable parameters are described in Table 2.

Table 1: Fixed and variable input parameters

	Soft soil	**Embankment**
Cohesion (C)	2 [kN/m^2]	1 [kN/m^2]
Internal friction angle (φ)	25 [degrees]	33 [degrees]
Dilatancy (ψ)	0 [degrees]	3 [degrees]
Pre-Overburden pressure (POP)	10 [kPa]	0 [kPa]
Power (m)	1 [-]	0,5 [-]
Oedometer stiffness (E_{oed})	variable, fixed relation with E_{50}	2,000E+04 [kN/m^2]
E_{50}	variable	1,500E+04 [kN/m^2]
Unloading-reloading stiffness (E_{ur})	variable, fixed relation with E_{50}	5,000E+04 [kN/m^2]

	Soft soil	**Embankment**
Dry volumetric weight (γ_d)	variable, fixed relation with E_{50}	18 [kN/m^3]
Wet volumetric weight (γ_w)	variable, fixed relation with E_{50}	20 [kN/m^3]

An assumption needs to be made regarding the volumetric weight of the soft soil. There is a relationship between the stiffness of the soil and its volumetric weight. Light soils like peat tend to have a low E modulus where denser soft soils like heavy clays have a relative high stiffness. This is a rather course approximation and does not hold for all soil types, but it is still better than the current design practice where such a relationship is not taken into account whatsoever. The set of variable parameters is given in Table 2.

Table 2: Sets of variable parameters in analysis

	γ (dry and wet)	E_{oed}	E_{50}	E_{ur}
Soil 1	11	700	1050	2100
Soil 2	12	1000	1500	3000
Soil 3	13	2000	3000	6000
Soil 4	14	3000	4500	9000
Soil 5	15	4000	6000	12000
Soil 6	16	5000	7500	15000

This results in six material sets with in essence only one variable, the stiffness of the soil. This varying stiffness will cause different one-dimensional settlements under a certain load and therefore act as one of the variables required in the ANN. These volumetric weights with their belonging elasticities and given loads result in vertical settlements between 7 centimeters and 2,5 meters. There are sufficient results between 10 centimeters and 2 meters of vertical settlement to be able to perform a good interpolation between the results.

To prepare a proper set of calculations, there needs to be variation in two other input variables the height of the embankment and the thickness of the soft soil layers. These are the geometrical variables in the analysis. The height of the embankment varies between 2 and 7 meters in steps of 1 meter. The thickness of the soft soil layers varies between 4 and 12 meters in steps of 2 meters. The 6 soil types together with the 6 embankment heights and the 5 thicknesses of the soft soil layers give a total of 180 calculations. In Figure 2, the model on which the calculations have been performed is drawn.

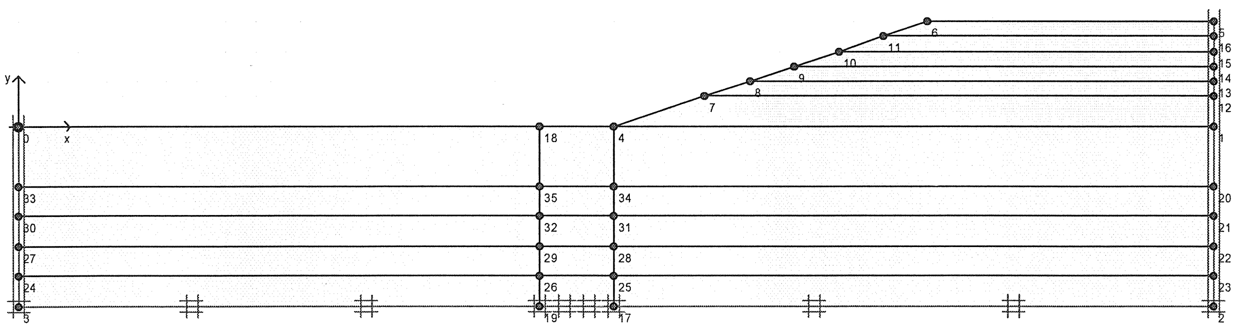


Figure 2: Geometrical model of the embankment

The slope of the embankment is kept constant at 1:3. The phreatic line is assumed to be 1 meter below surface level and all pressures are hydrostatic. There is no ditch next to the embankment.
The calculations have been performed and the results have been put into a database. In the next sections it will be described how this data can be put into an accessible tool.

6 Finding the best network

First, a network is trained to give the maximum horizontal deformations as output. The goal is to find a suitable network that gives enough accuracy with as few nodes (degrees of freedom) as possible. The number of layers in the network is fixed to two. One is theoretically not sufficient, and using three, will theoretically not improve the significance while it can very well cause overfitting. The number of nodes in both layers is varied. Four options have been investigated: two and three nodes per layer, both with and without a bias. Two or three nodes should be sufficient because the relationship is relatively straightforward. A network without a bias is an option because the input (0,0,0) should give as output also 0: a bias is therefore theoretically not required but it can improve the fit. One-noded feedforward networks have been used as well, but they give a Root Mean Squared Error (RMSE) of more than 5 percent. The RMSE as a function of the number of iterations for the different alternatives is shown in figure 3.

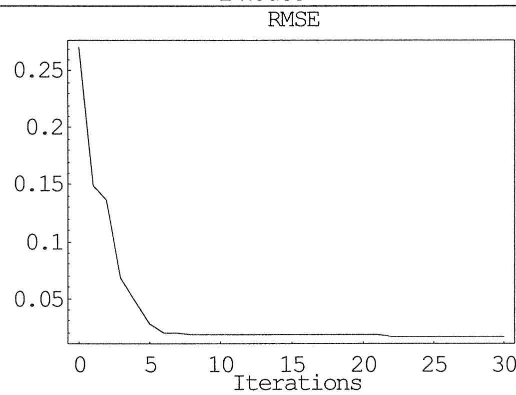
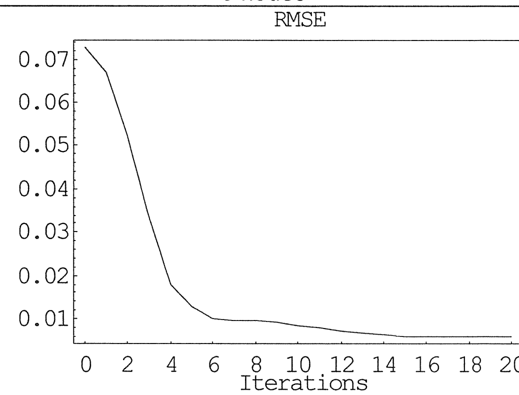
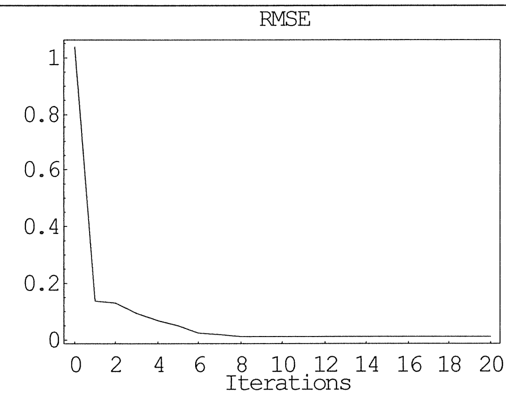
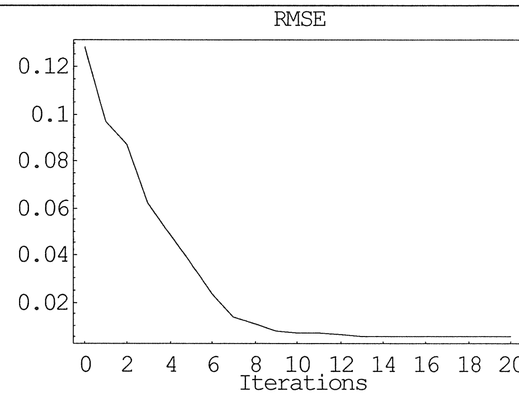
	2 nodes	3 nodes
no bias		
biased		

Figure 3: RMSE of the alternative models as a function of the number of iterations

The graphs show it is possible to reproduce the FEM results within one percent using only three parameters. Based on the graphs, it is decided to use a 2 noded biased feedforward network. When using 2 nodes without a bias, the RMSE remains structurally higher than with a bias. A further increase of the number of nodes only improves the significance marginally and it has the risk of overfitting the data and extrapolation of the results will lead to even more uncertainty.

To give an example of the output, a resulting equation belonging to the two noded biased network is given in Equation (11). It shows the horizontal deformations as a function of h, u_y and d

$$U_x = -2.19 + \frac{0.25}{1+e^{(1.12+0.107\times D-0.306\times H-1.15\times Uy)}} + \frac{2.43}{1+e^{(-1.98-0.018\times D+0.060\times H-0.636\times Uy)}} \qquad (11)$$

It can be concluded that an ANN can reproduce the results of this finite element analysis by using only three parameters within an error tolerance of 1 percent. Therefore, it can already be seen as a proof of principle and should therefore be extended to work with better finite element calculations and more nodes. This will be done in the next sections.

7 Comparison of the results

Eight different hypothetical cases have been calculated both with finite elements, the analytical method of “De Leeuw” (De Leeuw, 1963), and with the neural network to give an idea of the capabilities of the neural network. The results of the calculations are displayed in table 3.

Table 3: comparison of FEM, ANN and analytical results.

Horizontal deformation	FEM	ANN 2b	De Leeuw
Case 1	-0,1886	-0.1869	-0.2052
Case 2	-0,2658	-0.2660	-0.3420

Horizontal deformation	FEM	ANN 2b	De Leeuw
Case 3	-0,2858	-0.2835	-0.3078
Case 4	-0,4231	-0.4249	-0.5131
Case 5	-0,0651	-0.0653	-0.0684
Case 6	-0,0852	-0.0854	-0.1140
Case 7	-0,1084	-0.1035	-0.1026
Case 8	-0,1414	-0.1390	-0.1710

Clearly, the neural network can, using only three input parameters, reproduce the FEM results very well. The three parameters are clearly sufficient to distinguish the processes leading to the horizontal deformations in the FEM calculation. The analytical method of "De Leeuw" shows similar behavior to the FEM results, but tends to overestimate the deformations – which is a well-known feature of the method.

8 Introduction of pore water pressures

The model presented up to now serves to proof that is it possible to reproduce FEM results with only a few parameters to get a specific result. The performed FEM analyses thus far, are not a good representation of real embankments. It has been chosen initially to ignore the development of excess pore water pressures, because this depends for a large part on the scheme on which the embankment is raised by which time dependency comes into account. It is essential, though, that the build-up of pore water pressures is taken into account properly, because these excess pore pressures will lead to lower effective stress and therefore larger plastic deformations.

Different approaches have been implemented in order to implement pore water pressures and thereby the building history of the embankment. Fundamentally, the best way to take the excess pore pressures due to the previous building stage into account is to actually put them in the FEM calculations and thereby also in the ANN. The deformations that follow because of this are also calculated according to the chosen material model.

The main practical disadvantage is that the number of calculations required to train the ANN increases dramatically. However, because this is scientifically the most sound approach to predict the horizontal deformations, this approach is worked out hereafter.

8.1 Design with five parameters: Uy, H0, H1, D, and Uex

Two shortcomings of the initial method are overcome by using five parameters instead of the initial three. One extra parameter is the initial height of the embankment, H0. High embankments cannot be constructed at once, so the deformations will need to be calculated step by step. Any model that assumes the loading will take place instantaneously does not represent reality well.

Then, as a second extra parameter, the user needs to input the amount of excess pore water pressure in the soft soil layer before the embankment is raised. In reality, the excess pore water pressures after a consolidation phase are a complex function of depth. To keep it relatively simple, a percentage of the initial excess pore water pressures needs to be given, as if H0 was put there at once. For example, if half of the excess pore water pressure of the raise from surface level to H0 has dissipated, one needs to enter 50% for this variable.

One parameter that will be kept constant in the analysis is the cohesion of the soft subsoil. The choice which cohesion to use is, in contrast to what other analytic models suggest, very important. A high cohesion will make it easier to perform some calculations because failure will occur later. It will give too small deformations when an embankment on a soil with low cohesion reaches failure. One additional reason why the choice of the strength parameters is important, specifically when using the Hardening Soil model, is because of the fact that undrained calculations are performed. In an undrained analysis, one needs to make sure that the stress path during loading is realistic. Based on these considerations, a rather low value for the cohesion is chosen of 5 kPa. Wherever failure is obvious, the results are disregarded. This way, the network does give good results by taking plasticity into account, if an embankment with low cohesion is near failure. If the embankment has a high cohesion, the horizontal deformations will be overestimated if the results with low cohesion are extrapolated.

A disadvantage of this general approach is that there is a strong influence of the cohesion on the deformation when the embankment is near failure even though the cohesion is not a variable. It is an option to make the cohesion variable as well, but this will require even more calculations and make the tool less usable in an early stage of a project.

8.2 Time dependent calculations with pore water pressures

To predict the deformations using the FEM analysis, first, loading from H0 to H1 will take place under undrained conditions. Next, consolidation will take place until the maximum excess pore pressures have dissipated to less than 1 kPa.

The basic set of FEM calculations is the same as described in section 5. This were 180 calculations based on an analysis with three parameters. In the original analysis, the calculations have been performed with an initial excess pore water pressure of 0%. For this extended analysis, the same set of calculations have been run with an initial excess pore water pressure of 25%, 50% and 60% to cover the input space of Uex. To also cover the input space for H0 and H1, the same values for H1 have been preserved. Each H1 has been calculated with a H0 that is 1, 2 and 4 meters lower. This gives a theoretical total of 2160 time-dependant FEM calculations to train the ANN. The network has been trained with less values because many input combinations do not yield a stable embankment and a prediction of horizontal deformations is therefore not possible.

8.3 Results in time

After the calculations have been performed, the data has been extracted and the ANN has been trained, the results can be examined. As a first step, it has been chosen only to train the neural network to report the immediate undrained deformations and the deformations after consolidation. Given a permeability and the thickness of the soft soil layers, this can be calculated into a time-settlement curve. In the future, this can also be done by extracting more FEM data to train the network.

Once again, the ANN can reproduce the results of the FEM analysis well within an error margin of one percent. Using this ANN, it is easily possible to reproduce the building of an embankment in several loading steps while the complete building history of the embankment is taken into account.

Comparison with analytical methods is not very relevant because they do not take the building history into account. With any analytical method one will be able to find a construction scheme that fits the result.

9 Conclusions and Future developments

This approach is successful because one is able to reproduce the time dependent FEM behavior with sufficient accuracy. All of this with relatively simple input. There is much room left for improvement, though. The first aspect that needs to be addressed is that the parameters used, are rather difficult for early geotechnical design. It is not known to what extend the Uex has to decrease in order to make a new loading step unless a detailed stability analysis is made. Such a detailed calculation cannot be made early in a project. Several possibilities exist to overcome this issue. For now it has been chosen to replace Uex with the safety factor. They are related, but the safety factor is much better known in advance because there are strict guidelines for this factor.

The batch of calculations used in the previous sections has been chosen in such way that the input is spread well. This did cause many calculation results with a factor of safety below one, and therefore without a prediction of the horizontal deformations. In the future, the input space should be chosen in such a way that only calculations are run with a sufficient factor of safety. This will be much easier if this factor of safety is an input parameter.

Much has been learned in the project thus far for the next time a batch of calculations will be run. Very good care needs to be taken to the quality of the undrained calculations. It is very likely that another parameter set will need to be used for the drained and undrained parameters in order to simulate the right behavior.

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